FREQUENCY DOMAIN SYSTEM IDENTIFICATION OF A LIGHT HELICOPTER IN HOVER

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ABSTRACT

This paper presents the implementation of a Multi-Input Single-Output fully coupled transfer function model of a civil light helicopter in hover. A frequency domain identification method is implemented. It is discussed that the chosen frequency range of excitation allows to capture some important rotor dynamic modes. Therefore, studies that require coupled rotor/body models are possible. The pitch-rate response with respect to the longitudinal cyclic is considered in detail throughout the paper. Different transfer functions are evaluated to compare the capability to capture the main helicopter dynamic modes. It is concluded that models with order less than 6 are not able to model the lead-lag dynamics in the pitch axis. Nevertheless, a transfer function model of the 4^{th} order can provide acceptable results for handling qualities evaluations. The identified transfer function models are validated in the time domain with different input signals than those used during the identification and show good predictive capabilities. From the results it is possible to conclude that the identified transfer function models are able to capture the main dynamic characteristics of the considered light helicopter in hover.

INTRODUCTION

At the Max Planck Institute for Biological Cybernetics the interaction is investigated between a pilot with limited flying skills and augmented vehicles that will be part of a new concept of personal air transport systems (Ref. 1). This study will provide contributions to create a vehicle that is as easy to fly as it is to drive a car. This project is focused on light helicopters as these best reflect the properties of a vehicle that could be used in a personal air transport system. The flight state of interest throughout the project is hover, since it is commonly considered one of the most difficult to perform for a non-expert pilot. The goal of the project is to study which augmented system features allow a pilot with limited flying skills to reach similar performance as a highly-trained pilot. The project is composed of three main phases. The first phase is the identification of a rigid body dynamic model of a light helicopter. The second phase represents the realization of augmented systems for the model identified. The third phase consists of handling qualities and human performance evaluations in piloted closed loop control tasks, with and without the augmented systems. For the three phases the MPI CyberMotion Simulator (CMS) will be used, see Fig. 1. The 8 Degrees Of Freedom (DOF) robotic arm has a large motion envelope and is well suited to simulate the identified helicopter model in order to study the effects of augmentation techniques on non-expert pilots control performance.

This paper focuses on the first phase of the project, particularly on the implementation of a MISO (Multi Input Single Output) fully coupled transfer function model of a light helicopter in hover. Such helicopter model could be used for different possible applications, such as developing control-systems, making pilot handling-qualities evaluations in simulations, evaluating the fidelity of visual- and motion-systems of simulators and training pilots (Ref. 2). The implementation of a system identification model was preferred to a non-linear full-flight-envelope model. This choice relies on studies which demonstrated the deficiencies of a complex non-linear model in predicting some fundamental dynamics (Ref. 3). Indeed, dynamics like primary roll, vertical response

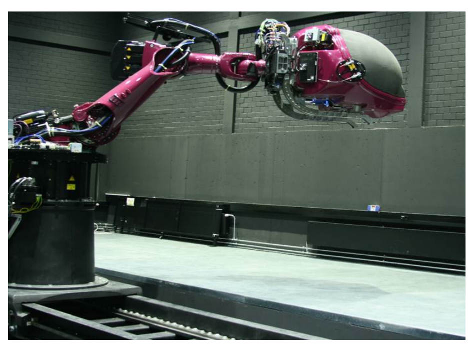

Fig. 1 The 8 DoF MPI CyberMotion Simulator (http://www.cyberneum.de/).

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or pitch/roll cross-coupling may not be correctly captured if the model is implemented to be valid over the full-flight-envelope. Data collected for a specific condition can provide system identification models which have been proven to give better results. However, so far system identification for civil purposes has not been common in the helicopter field because of, e.g., expensive instrumentation usually used for military purposes, unavailability of multiple hours of flight tests and lack of interest from some civil companies in system identification studies. Therefore, identifying a civil helicopter model represents one of the main challenges of the project. The identification method implemented in this study is based on the frequency domain techniques developed in the last few decades and applied in many rotorcraft works (Refs. 4, 5).

An important aspect analyzed in this paper is the choice of the model dynamic complexity. A 6 DOF model is generally adequate for handling qualities evaluations. However, higher order model structures are necessary for applications like simulation validation or flight control system design (Ref. 6). Many works demonstrated that high bandwidth control systems for helicopter need to include rotor degrees of freedom (Ref. 7). Tischler investigated high order mathematical models and proved that for a hingeless single-rotor helicopter the coupled body/rotor-flapping mode limits the gain on attitude control feedback, while the lead-lag mode limits the gain on attitude-rate control feedback (Ref. 8). A variable-stability CH47 helicopter was used in (Ref. 9) to demonstrate how rotor dynamics and control system lags can influence the feedback gain limits. Recent studies by DLR in Germany have shown the importance of suppressing the air resonance due to the regressive lead-lag mode and particularly visible for high feedback gains in closed-loop controllers (Ref. 10). These studies suggest that considering rotor's DOF can be necessary to implement augmented control systems and to analyze their differences, which is one of the main goals of the current project. For this reason, model complexity analysis will be done to assess whether the identified transfer function models can capture body/rotor couplings. Furthermore, model reliability will be evaluated for handling qualities studies and for control system design.

This paper will present results on:

- implementing a SISO non-parametric model of a civil light helicopter in hover;
- conditioning the responses to consider the effect of secondary inputs and applying the composite windowing method;
- implementing a MISO parametric model;
- analyzing the model complexity;
- validating the identified MISO parametric models in the time domain with different input signals than those used during the identification.

DATA COLLECTION FOR IDENTIFICATION

The data for system identification were collected during two test flights, each with a duration of one hour. A Robinson R44 Raven II was used, which is a light helicopter with a single engine, a semi-rigid two-bladed main rotor and a two-bladed tail rotor. Several piloted frequency sweeps were recorded for each control axis to ensure the identification of the parametric transfer function models. Doublets and steps were also recorded to allow the validation of the identified models in the time domain with different input maneuvers. A preliminary training phase was necessary before and during flight to ensure that the pilot could safely and correctly perform the maneuvers of interest.

Two Global Positioning System antennas and an Inertial Measurement Unit were used that consisted of Fiber Optic Gyros and Micro Electrical Mechanical System accelerometers. These instruments allowed collecting the signals defined as the outputs of the model to be identified: the position of the helicopter with respect to the inertial frame, the attitude, the angular rates and the linear accelerations. Four optical sensors were chosen to measure the input signals from the pilot (two for the cyclic stick deflections, one for the collective lever, and one for the pedals). A sample rate of 100 Hz was chosen for all signals. A frequency range of excitation between 0.3 and 17 rad/s was considered during the piloted sweeps. The details of the flight tests are presented in (Ref. 11)

In Fig. 2 the time data of two concatenated frequency sweeps are shown for the longitudinal axis. Each frequency sweep has a duration of about 100 seconds. The sweep maneuvers start in hover with a few seconds of trim and end with the same initial trim condition. The longest period of the sweeps is of about 20 s which corresponds to a frequency of about 0.05 Hz. Then the pilot slowly increases the frequency of the sweep till the period of 0.4 s is reached (≈ 2.5 Hz). In the figure the primary helicopter responses to the longitudinal stick deflection are also shown. It can be seen how the variation of the pitch rate reaches a maximum of about 20 deg/sec. This size of excitation ensures the identification of models which are also accurate for maneuvers with large excursions. The high frequency content visible in the pitch rate (q) is not present in the longitudinal translational velocity (u) and in the pitch angle (θ) where only low frequencies are generally involved. This high frequency content is important for the model complexity analysis involving body/rotor couplings. For this reason, the pitch rate response q/δ_{long} will be considered in detail throughout the paper. Concatenating two or more frequency sweeps as seen in Fig. 2 allows to obtain a rich spectral content over the frequency range of interest. Therefore the same procedure was applied to the other control axes.

IDENTIFICATION

In the past few decades system identification in the rotorcraft field has grown considerably. This is probably due to a better

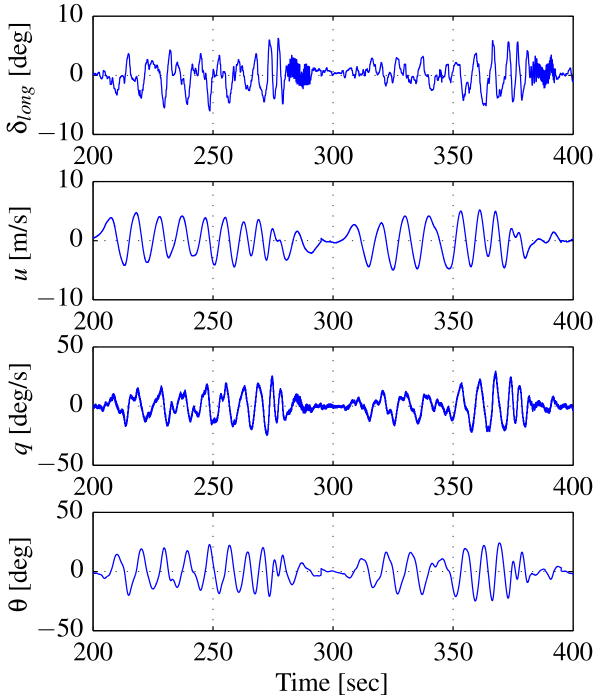

Fig. 2 A frequency sweep in the longitudinal axis in hover. δ_{long} = cyclic longitudinal deflection, u = longitudinal translational velocity, θ = pitch angle, q = pitch rate.

understanding of system identification theories and to novel applications of identified helicopter models such as control system design (Ref. 12). Among the different system identification methods the one proposed by Tischler in (Ref. 13) has been applied for many helicopter studies. Very good results were obtained that confirmed and validated the reliability of this method. The benefits derived from applying this method are mainly due to the fact that it is frequency based. This means that a non-parametric system identification phase can be implemented which allows to get some preliminary information about the model structure. Moreover, frequency responses conditioned by multiple partially correlated inputs can be computed. Finally, procedures like composite windowing and tools like the coherence function can be used to improve and analyze the results. For these reasons, this method was considered well suited for the purpose of the current study. Many system identification works based on this method were implemented by using the Comprehensive Identification from FrEquency Responses (CIFER®) software package. However, the results presented in this paper were obtained by implementing the method with the numerical computation environment MATLAB®. An important step of the identification process is to determine the complexity of the model and, therefore, which dynamics need to be included. This generally depends on the frequency range of interest and on the number of dynamical states measured during the flight tests (Ref. 14). For the present project the collection of data did not include any direct measurement of rotor-state dynamics. Nevertheless, it will be shown how the frequency range exited during the test flights allows to capture some rotor dynamic modes. Therefore, the implementation of higher order rotor/body models is possible.

Non-parametric SISO and MISO identification

The first step of the considered system identification method is the generation of a frequency response database containing the entire set of nonparametric single-input single-output frequency responses. These responses are obtained by concatenating two or three frequency-sweep time histories collected during the flight tests, as shown in Fig. 2. Then, the Fast Fourier Transform is applied by using overlapped-windowed spectral averaging. An example is shown in Fig. 3 with the bode plot of the pitch rate response estimation to the longitudinal stick deflection (q/δ_{long}). Here a window of 20 seconds was used. The figure shows also the coherence function which is an important tool at this stage since it permits assessing the goodness of the collected data (Ref. 15). This function can assume values between zero and one and indicates the fraction of the output spectrum linearly related to the input spectrum. A decrease in the coherence function can be due to process noise, nonlinearities, lack of input excitation or lack of rotorcraft response. Generally, coherence values of 0.6 and above are considered acceptable (Ref. 16). In Fig. 3 it can be seen that the q/δ_{long} frequency response shows a good level of coherence within the considered frequency range of excitation (0.3-17 rad/s).

However, using only one window increases the accuracy over a limited range of frequencies. This can be noticed in Fig. 3 where at high frequencies there is an increase in the random error that is reflected in the magnitude and the phase oscillations, even though the coherence function remains above the boundary of 0.6. Choosing a smaller window would reduce the random error but introduces a loss of accuracy at lower frequencies. To overcome this issue a procedure called composite windowing can be used during the nonparametric identification. In this procedure the conditioned frequency responses are computed with windows of different size. Then, a weighted nonlinear Least-Squares minimization method is implemented that provides the composite-conditioned frequency responses. The new responses calculated in this way are characterized by a good coherence and low random error over the entire frequency range of interest. Usually, the choice of the windows is done by considering the minimum and the maximum frequency desired for the identification (Ref. 17). For the present study a window of 40 seconds and one of 7 seconds were selected respectively as the largest and the smallest one. Three more windows were evenly distributed between these two for a total of five different window sizes.

Helicopter system identification studies requires that input-output couplings and multiple partially correlated inputs effects are taken into account in order to compute the actual SISO frequency responses. The multi-input identification technique allows to consider these effects by means of the

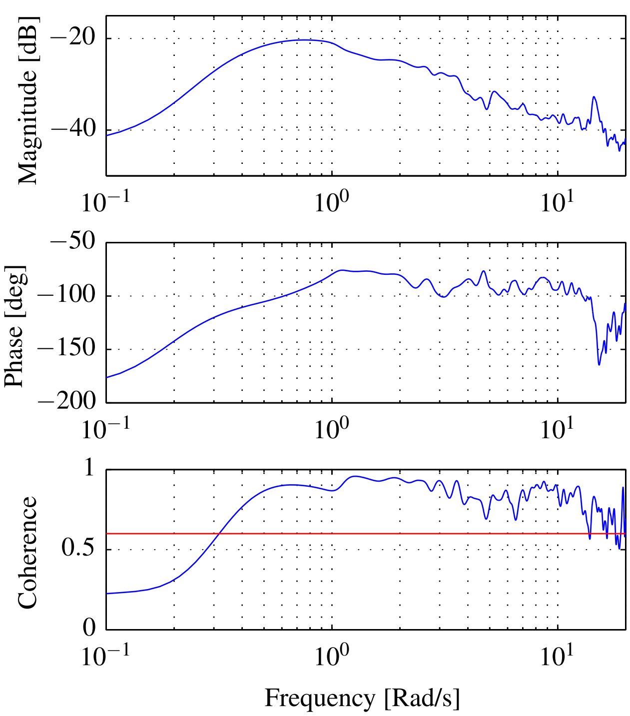

Fig. 3 Pitch axis SISO frequency response q/δ_{long}.

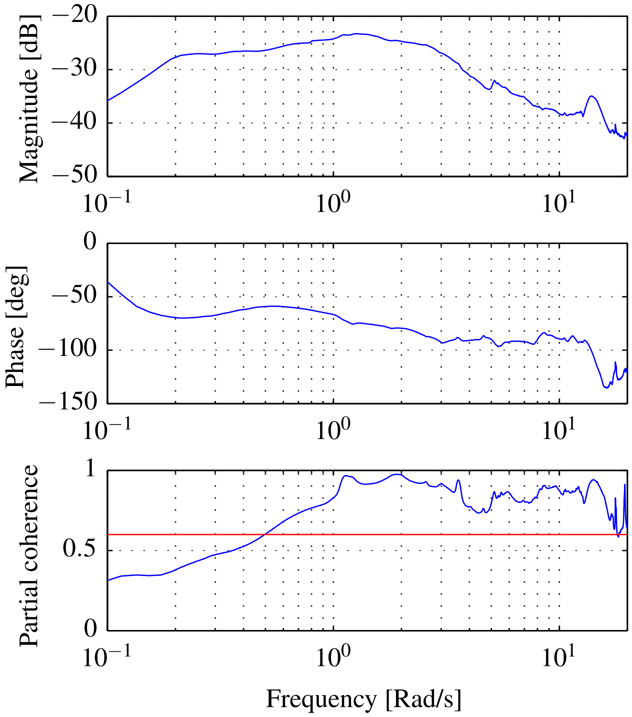

Fig. 4 Pitch axis MISO conditioned frequency response q/δ_{long} with composite windowing method applied.

so-called conditioned auto- and cross-spectra (Ref. 15). These allow to compute the conditioned frequency responses and the related partial coherences. The application of the conditioned-spectra and the composite method generated a set of 36 input-output conditioned frequency responses and the associated partial coherence functions. This result represent the final step of the non-parametric system identification process. An example of conditioned frequency response obtained by applying the composite windowing method is shown in Fig. 4 for the pitch-rate response q/δ_{long}. It is possible to notice how the response has a different shape with respect to the one in Fig. 3 due to the subtraction of the partially correlated inputs effects. Furthermore, the accuracy increases at higher frequencies due to the composite windowing procedure. The application of these methods makes much easier to recognize some important dynamical characteristics. At about 0.7 rad/sec the influence of the unstable phugoid mode appears while the resonance at about 14 rad/sec is due to the lightly damped regressive lead-lag mode. Similar analysis were considered for the other on- and off-axis conditioned frequency responses in which the effects of some rotor dynamic modes appeared as well as for the pitch response. From these evaluations it was possible to conclude that the considered frequency range of excitation is large enough to capture some important rotor dynamic modes.

Transfer function modeling identification

Once all the input-output conditioned frequency responses have been computed, two possible parametric system identification methods can be considered: the transfer function and the state-space modeling identification. In this paper the transfer function modeling procedure is implemented since it provides useful information on the fundamental dynamic characteristics and allows many key applications, such as handling qualities analysis and flight dynamic modeling for control system design (Ref. 18). Furthermore, comparing the matching quality of transfer-function models that differ in complexity can give information concerning the order of the system, the level of couplings and initial reasonable values for some parameters that could later be used for building a state-space identification model (Ref. 17). Transfer function models are obtained by fitting individual input-output frequency responses through the minimization of magnitude and phase errors. A detailed description of this approach is given in (Ref. 13). Different models were selected to fit the data response of each input-output axis over the selected frequency range (0.3-17 rad/s). The choice of the models was done in order to ensure a physical meaning and to avoid any over-parameterization that could lead to poor predictive capabilities.

The pitch-rate response to longitudinal stick input q/δ_{long} will be now considered in detail. Different models were selected to fit the frequency response and to adequately capture the main pitch dynamic modes. The first transfer function model considered is a coupled body/rotor 6^{th} order model used to represent the effects of the phugoid, the short period and the regressive lead-lag modes. The transfer function model and the relative fit cost is presented in Table 1. It can be noticed that a cost function is obtained well below 100, commonly considered in literature as a limit to ensure satisfactory accuracy.

Table 1 Pitch response transfer function models

Model	Transfer function^a	Fit cost
6$^{\text{th}}$ order	$\frac{0.11(3.928)[-1,0.327][0.213,14.265]e^{-0.019s}}{[-1,0.683][0.93,2.065][0.1,14.336]}$	6.17
4$^{\text{th}}$ order	$\frac{0.14(1.317)[-1,0.361]e^{-0.023s}}{[-1,0.733][0.947,1.33]}$	37.94

^{a}Shorthand notation: $[\xi,\omega]$ indicates $s^2+2\xi\omega s+\omega^2$, ξ damping ratio, ω natural frequency (rad/sec); (1/T) indicates s+(1/T), rad/sec

This result is reflected in the bode plot in Fig. 5 where it can be seen how the transfer function response follows the measured data with good accuracy. In this transfer function model each dominant dynamic mode is represented with a pair of complex conjugate poles. For the unstable phugoid mode the complex poles are located at 0.683 rad/sec as it was expected from the non-parametric MISO identification. Another pair of highly damped complex poles at 2.065 rad/sec allows to model the short period. Its effect is visible in Fig. 5 with a decrease of the magnitude and the phase. The rotor lead-lag mode effects are modeled with two complex poles at 14.336 rad/sec as also predicted in the non-parametric MISO phase. The effect of this mode is recognizable in the magnitude plot with a relevant peak and in the phase plot with a roll off. Finally, a residual equivalent time delay of 0.019 seconds is added to represent the effective delays related to sensor filtering, linkage dynamics between the stick and the rotor and additional non-modeled high frequency rotor dynamics. The error function plot shown in Fig. 6, is computed as difference in magnitude (dB) and phase (deg) between the real frequency response and the estimated response. The error is compared with the mismatch boundaries defined in the MIL-STD-1797 (Ref. 13). These boundaries represent the Maximum Unnoticeable Added Dynamics (MUAD) limits. When they are exceeded a pilot can detect a divergence in the modeled aircraft response characteristics (Ref. 19). They have been considered to evaluate the effects of unnoticeable dynamics in many helicopter identification studies (Refs. 6, 20). As can be seen in Fig. 6, the error is well within the boundaries for the entire frequency range of interest, which means that a pilot would consider the model responses almost indistinguishable from the actual flight response. This is generally considered a good starting point for handling-qualities analyses. Furthermore, it can be also concluded that this model is well suited for augmented control system design due to its ability to capture rotor DOFs relevant for this purpose.

A second 4$^{\text{th}}$ order transfer function model was considered for comparisons to the 6$^{\text{th}}$ order model. The 4$^{\text{th}}$ order model transfer function shown in Table 1 is based on the theory presented in (Ref. 21) where the classic fuselage longitudinal modes (phugoid and short period) are modeled with two pairs of complex poles whereas the rotor mode effects are included as equivalent time delay. The phugoid mode is represented with a pair of complex poles at 0.733 rad/sec which again confirms the effect of this mode at around 0.7 rad/sec as expected from the non-parametric MISO identification. The fuselage short period mode is modeled with another pair of

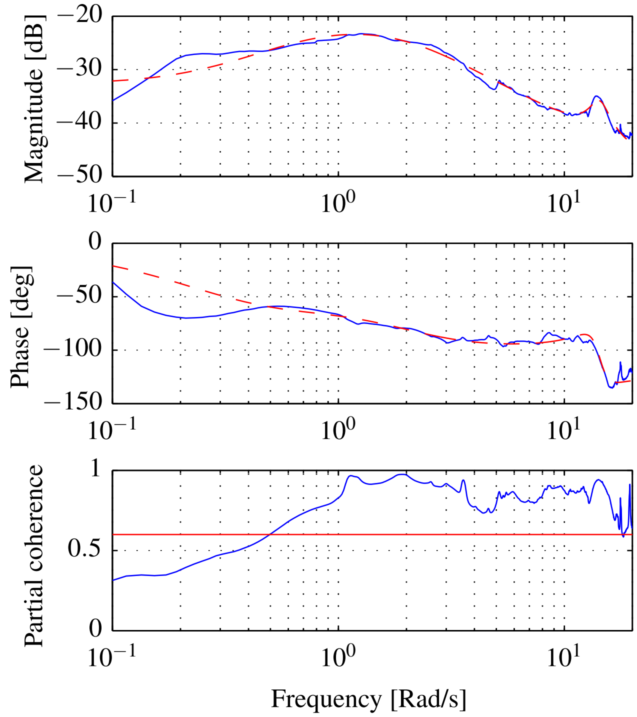

Fig. 5 Bode plot of the transfer function 6$^{\text{th}}$ order model for the pitch response q/δ_{long}. Flight data in continuous line. Model data in dashed line.

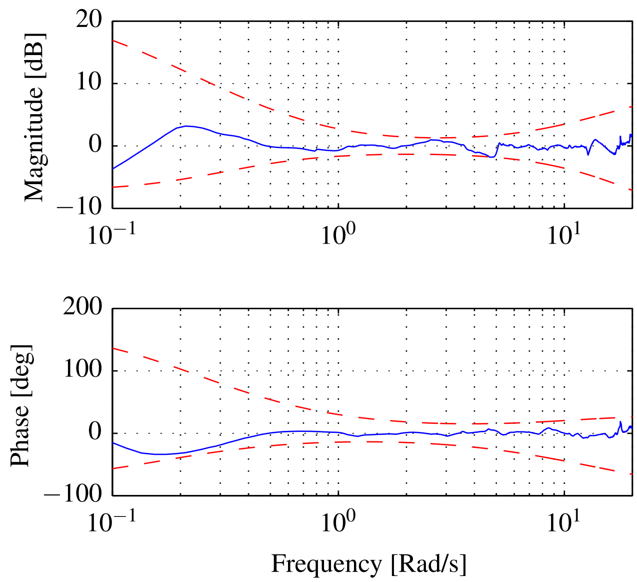

Fig. 6 Error transfer function of the q/δ_{long} response for the 6$^{\text{th}}$ order model with MUAD boundaries.

complex poles at 1.33 rad/sec. Furthermore, a larger equivalent time delay is necessary than the one used for the 6$^{\text{th}}$ order model to account for residual high frequency rotor dynamics. As can be seen in Fig. 7 the reduced order model still presents good accuracy over the entire frequency range but it is not able to adequately capture the high frequency lead-lag mode effects. For this reason the associated cost function shown in Table 1 is higher than for the 6$^{\text{th}}$ order model but still within the guideline boundary. In Fig. 8 it can be also noticed how

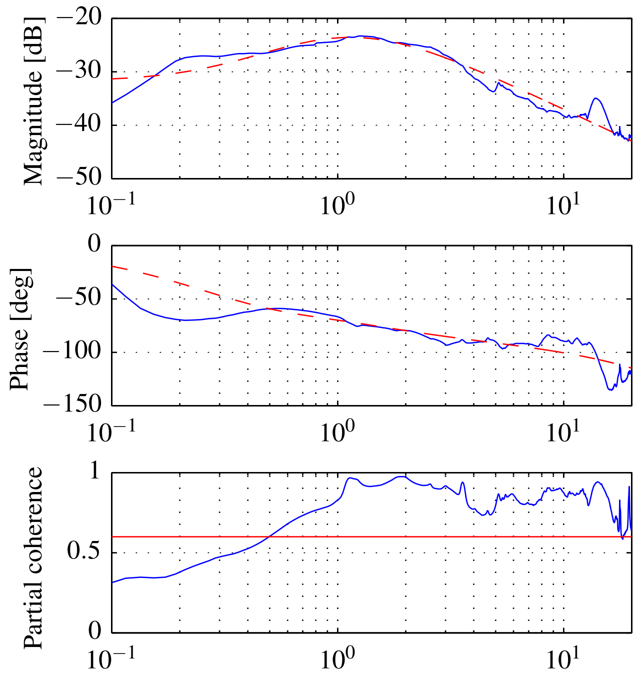

Fig. 7 Bode plot of the transfer function 4$^{\text{th}}$ order model for the pitch response q/δ_{long}. Flight data in continuous line. Model data in dashed line.

the error increases at higher frequencies due to the fact that the rotor dynamic modes are not modeled. Nevertheless, it remains within the MUAD mismatch boundaries. Therefore, it is possible to conclude that the 4$^{\text{th}}$ order transfer function model is well suited for handling-qualities analyses, but its incapability to capture high frequency rotor modes makes it inadequate for control system design. Other lower order models were considered to fit the pitch response but none of them were capable to represent the main dynamic modes of the longitudinal response since cost functions were well above the guideline limit of 100 and error functions were outside the MUAD limits.

Further transfer function models that are associated with responses considered relevant for hover and low-speed flight are presented in APPENDIX A. These include the dominant angular-rate responses to the cyclic and pedal control inputs (q/δ_{long}, p/δ_{lat}, r/δ_{ped}) and the vertical axis acceleration to the collective input (a_z/δ_{col}). The described transfer function

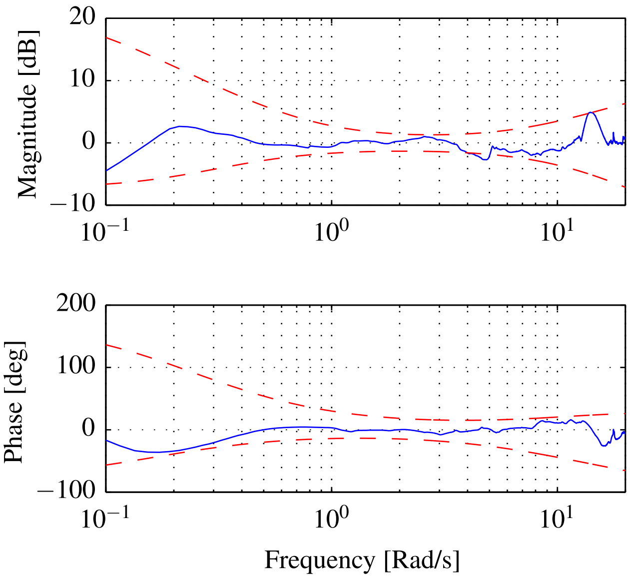

Fig. 8 Error transfer function of the q/δ_{long} response for the 4$^{\text{th}}$ order model with MUAD boundaries.

models are capable of capturing the rotor modes important for control system design. For these identified models, cost function values less than 100 were obtained, which means that they adequately represent the dynamics of the chosen helicopter in the considered hover condition. This is a favorable result considering that only two hours of test flights with a non-experienced test pilot were sufficient to collect reliable data for system identification studies.

MISO model time-domain validation

The final step of the MISO system identification presented in this paper involves the time domain validation of the identified parametric models. This method is implemented by using different maneuvers (doublets or steps) from those used during the identification process (frequency sweeps). The method is based on the theory proposed by Tischler in (Ref. 13). Again the pitch axis is considered in detail. As the transfer function models contained an unstable phugoid mode at around 0.7 rad/sec, the simulations used for validation can easily diverge from the measured helicopter response. Therefore, a time record of a few seconds is considered to evaluate the identified pitch model. Obviously, the maneuver performed by the pilot involved the use of all the controls to maintain the helicopter stability. Therefore, also the small contribution of the other inputs is taken into account for the validation. Fig. 9 shows the result for the q/δ_{long} response. The 6$^{\text{th}}$ and the 4$^{\text{th}}$ order models responses are indistinguishable. The main control input δ_{long} is shown over time and the model responses are compared with the measured one. The fit error of both models was approximately 1.4 and the Theil Inequality Coefficient (TIC) approximately 0.06 and these metrics are considered in literature to evaluate the goodness of the model prediction (Ref. 22). A fit error less than 2.0 usually represents

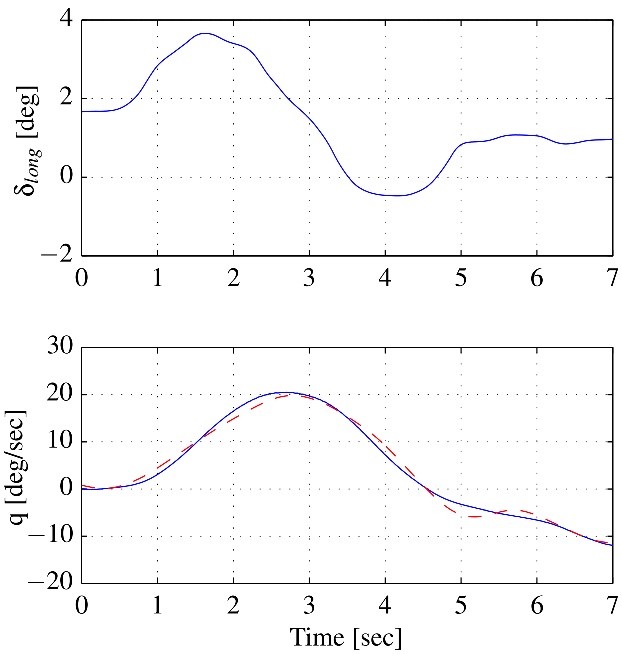

Fig. 9 Time domain 6th and 4th order model verification for longitudinal response q/δ_{long}. Flight data in continuous line. Model data in dashed line.

acceptable level of accuracy. The TIC is instead a normalized criterion. A TIC equal to zero means perfect predictive capability while a TIC equal to one means no predictive capability. Similar results are also obtained from the other responses which are detailed in APPENDIX B.

CONCLUSIONS

This paper has presented results on the implementation of a fully coupled transfer function model for a civil light helicopter in hover. From the obtained frequency responses, it was deduced that the chosen frequency range of excitation allows to capture some important rotor dynamic modes. Therefore, studies can be done that require coupled rotor/body models like control system design. The pitch-rate response with respect to the longitudinal cyclic was considered in detail and it was concluded that transfer function models with order lower than 4 cannot satisfactorily fit the main longitudinal helicopter modes since cost functions and Maximum Unnoticeable Added Dynamics boundaries are exceeded. Furthermore, it was concluded that a 6th order transfer function model is necessary to properly capture the rotor high frequency lead-lag mode needed to implement control system design studies. However, a 4th order transfer function model can already provide acceptable results for handling qualities studies. Finally, the identified transfer function models were validated in the time domain with different input signals than those used during the identification. Both models provided good predictive capability as expected from the results obtained in the frequency domain.

The results presented in this paper represent a starting point for the development of a state-space identification model. After that, the fully coupled state-space model will be used to implement different augmented control systems. The last step of the project consists of the validation of the augmented systems through handling qualities and human performance evaluations in piloted closed loop control tasks performed with the MPI CyberMotion Simulator.

APPENDIX A

In this appendix, the transfer function models for the dominant angular-rate responses to the cyclic and pedal control inputs (q/δ_{long}, p/δ_{lat}, r/δ_{ped}) and the vertical axis acceleration to the collective input (a_z/δ_{col}) are presented. The bode plot of the transfer functions fitted on the conditioned frequency responses and the relative error functions are shown in Figs 10-15. The obtained models and the cost functions are indicated in Table 2. For each response the dominant modes are chosen based on the models considered in literature. In Fig.10 the p/δ_{lat} lateral response shows a similar lead-lag mode to the q/δ_{long} longitudinal response and the same pair of complex poles at 14.336 rad/sec is considered. Furthermore, the dutch-roll and the lateral-flapping modes are modeled with two pairs of complex poles at 1.76 and 13.9 rad/sec respectively. In Fig. 12 the a_z/δ_{col} frequency response shows the coning inflow effect with a high frequency peak in the magnitude plot. This phenomenon is particularly visible in hover when the collective position rapidly changes. The coning inflow effect is approximated by adding a zero at 10.34 rad/sec and a time delay ($e^{-0.028s}$) to the 1st order model used to represent the heave mode. Finally, the r/δ_{ped} frequency response is presented in Fig.14. Here, the yaw damping mode is modeled with a second order system. The error function plots in Figs. 11, 13 and 15 show good results for the p/δ_{lat} and the a_z/δ_{col} responses but for the considered r/δ_{ped} model the error function exceeds the boundaries for frequencies below 2 rad/sec.

Table 2 Transfer function models main hover responses

Model	Transfer function[a]	Fit cost
p/δ_{lat}	$\frac{0.338(0.98)[0.21,16.6][0.37,12.1]e^{-0.001s}}{[0.1,14.336][0.97,1.76][0.8,13.9]}$	34.16
r/δ_{ped}	$\frac{0.02[1,7.85]}{[1,2.52]}$	95.23
a_z/δ_{col}	$\frac{0.05(0)(10.34)e^{-0.028s}}{(0.24)}$	15.98

[a]Shorthand notation: $[\xi,\omega]$ indicates $s^2+2\xi\omega s+\omega^2$, ξ damping ratio, ω natural frequency (rad/sec); (1/T) indicates s+(1/T), rad/sec

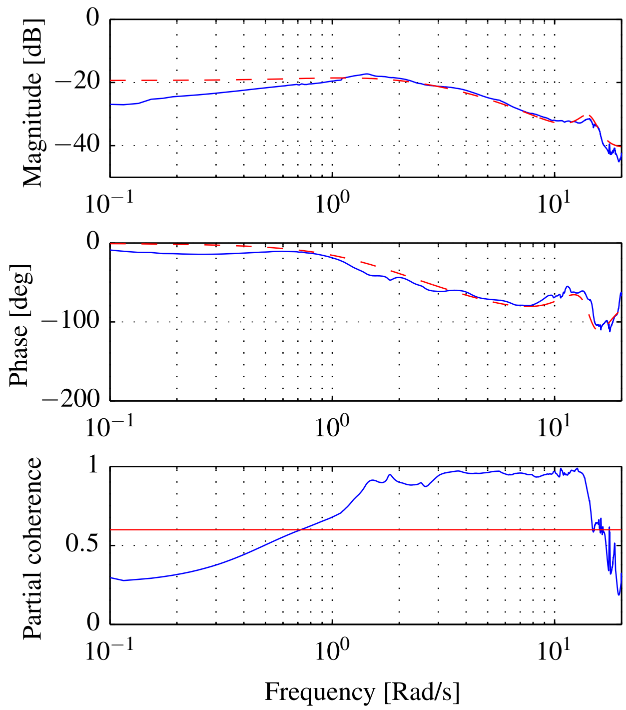

Fig. 10 Transfer function bode plot roll response p/δ_{lat}. Flight data in continuous line. Model data in dashed line.

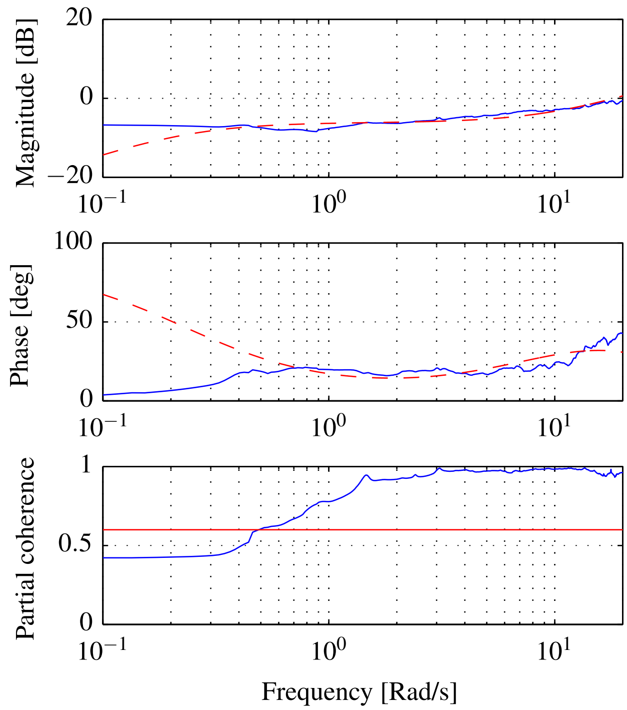

Fig. 12 Transfer function bode plot heave response a_z/δ_{col}. Flight data in continuous line. Model data in dashed line.

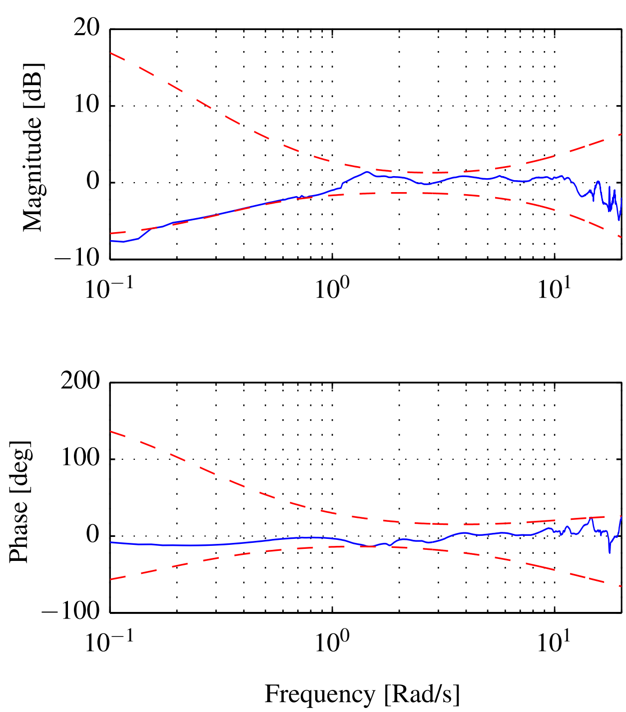

Fig. 11 Error transfer function of the p/δ_{lat} response with MUAD boundaries.

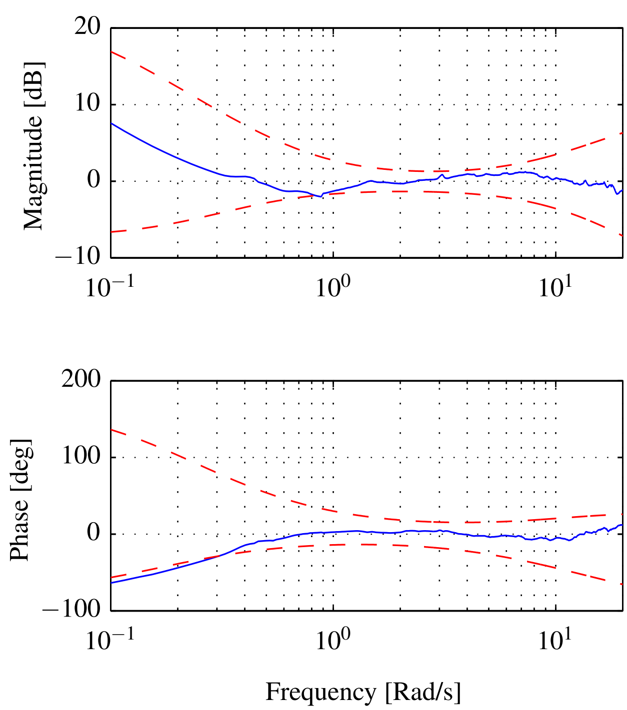

Fig. 13 Error transfer function of the a_z/δ_{col} response with MUAD boundaries.

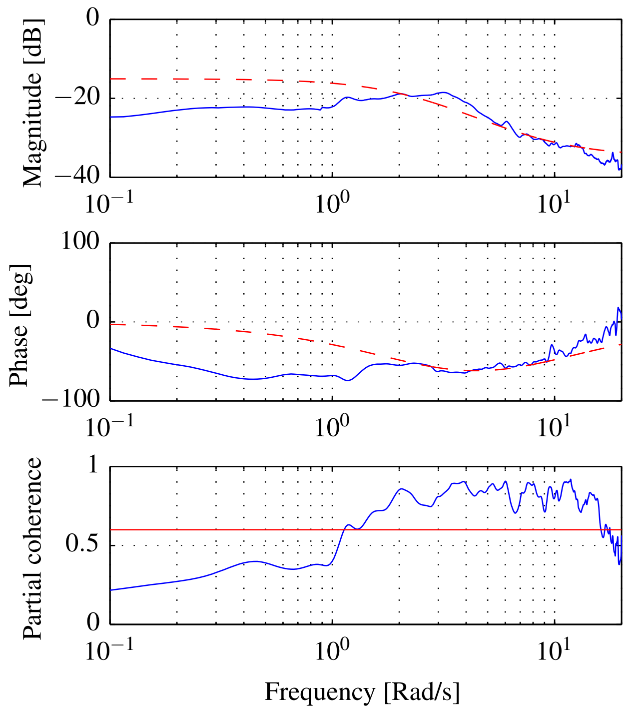

Fig. 14 Transfer function bode plot yaw response r/δ_{ped}. Flight data in continuous line. Model data in dashed line.

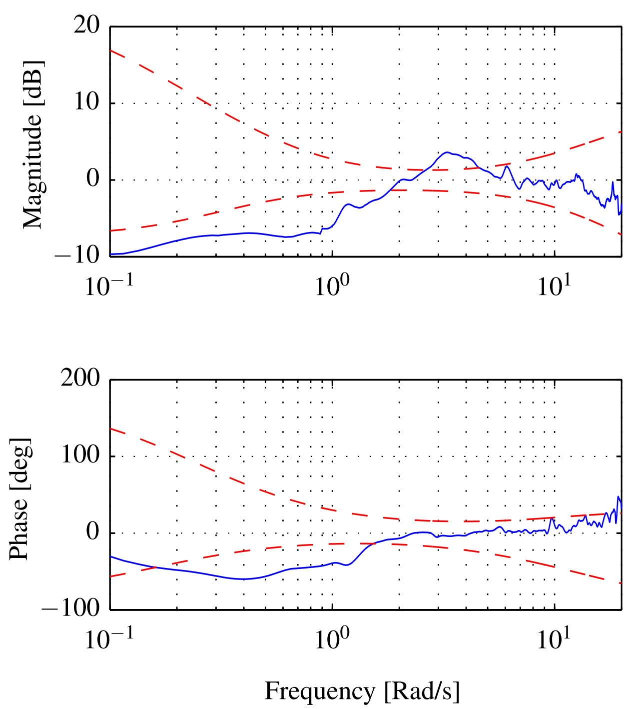

Fig. 15 Error transfer function of the r/δ_{ped} response with MUAD boundaries.

APPENDIX B

The time domain validation results for the main on-axis responses in hover (lateral response p/δ_{lat}, heave response a_z/δ_{col} and yaw response r/δ_{ped}) are shown in Figs 16-18. For the heave a_z/δ_{col} response validation a high frequency pole is added to the transfer function model in Table 2 to ensure physical causality. Different kinds of maneuvers (doublets, steps) are considered from those used during the system identification process (frequency sweeps). As can be noticed in the figures, good level of accuracy is achieved for all the responses.

For the p/δ_{lat} response in Fig. 16 a fit error function is obtained of 0.04 and a Theil Inequality Coefficient (TIC) of 0.045. For the a_z/δ_{col} response in Fig. 17 a fit error function is obtained of 0.05 and a Theil Inequality Coefficient (TIC) of 0.07. For the r/δ_{ped} response in Fig. 18 a fit error function is obtained of 1.8 and a Theil Inequality Coefficient (TIC) of 0.063. These metrics indicate a good level of accuracy.

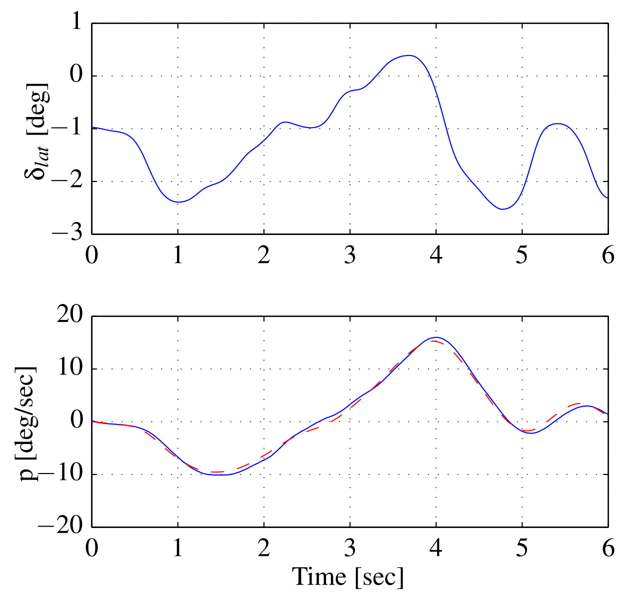

Fig. 16 Time domain verification of the lateral response p/δ_{lat}. Flight data in continuous line. Model data in dashed line.

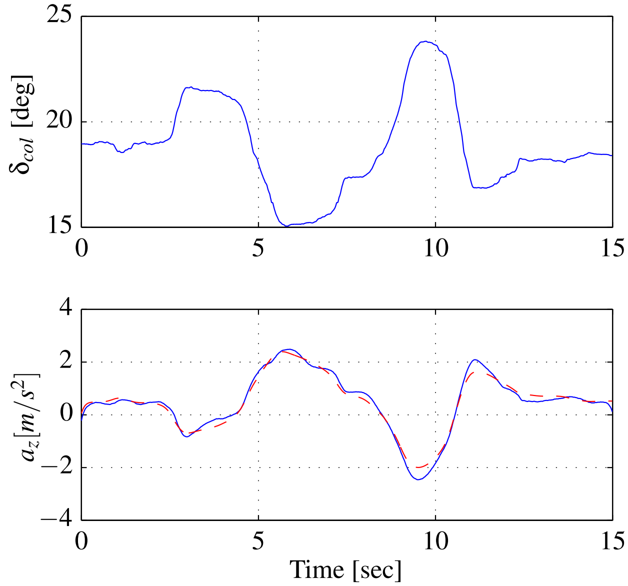

Fig. 17 Time domain verification of the heave response a_z/δ_{col}. Flight data in continuous line. Model data in dashed line.

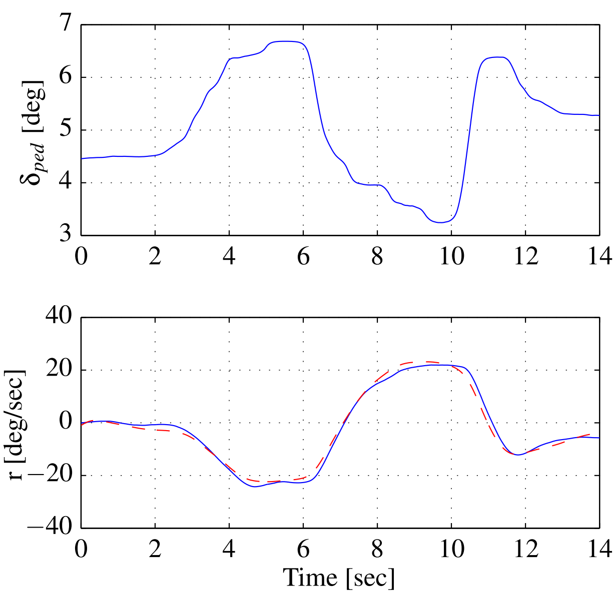

Fig. 18 Time domain verification of the yaw response r/δ_{ped}. Flight data in continuous line. Model data in dashed line.

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